Statistics

Lecture 16

Feb 19-8:47 AM

Working with claims $S6 24-26$

claim could be about any parameter.

It could be about

1) Pop. Prop. P

2) Pop. Mean μ

3) Population standard deviation σ

our objective is To test that claim.

why do we test?

Because we want to determine whether claim is valid or claim is invalid.

If we conclude that   ~~Fail-to-Reject~~

claim is valid then we support it.

claim is invalid then we reject it.

Possible error

Valid claim but we reject it.

Invalid claim but we support it

~~Fail-to-Reject~~

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with every testing, there is a number α
$0 < \alpha < 1$ and it is called Significant level.

If α not given $\Rightarrow$ we use .05

Testing Methods:
1) Traditional Method
2) P-Value Method ✓
3) Confidence interval Method

Regardless of method used, Final Conclusion will be the Same.

How To express Final Conclusion
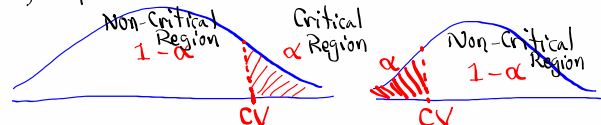   claim is valid $\Rightarrow$ Fail-to-Reject
   claim is invalid $\Rightarrow$ Reject the claim

Action \ Claim	Valid	Invalid
FTR Support	✓	Error
Reject	Error	✓

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Testing Types:
1) Right-Tail Test RTT   2) Left-Tail Test LTT

Non-Critical Region $1-\alpha$ Critical Region α α Non-Critical Region $1-\alpha$
CV   CV

3) Two-Tail Test TTT

CR $\alpha/2$ NCR $1-\alpha$ CR $\alpha/2$
CV   CV

Suppose $\alpha = .02$, RTT

NCR .98   CR .02
CV

Suppose $\alpha = .1$, TTT

CR .05   NCR .9   CR .05
CV   CV
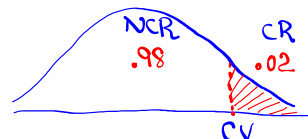
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Testing Process:

1) Set-up H_0 & H_1 ← Alternative Hypothesis H_a

H_0 → Null Hypothesis

2) Find all Critical values
   Drawing, Labeling, Shading, Full TI command

3) Find Computed Test statistic (CTS) and P-value (P).   Formula or Full TI Command needed.

4) Use Testing chart to determine the validity of H_0 and H_1.

5) Draw Final Conclusion about the claim.
   Claim could be H_0 or H_1.
   H_0 valid ⟺ H_1 invalid
   H_0 invalid ⟺ H_1 valid

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More on H_0 & H_1

H_0 must contain = Sign.

H_1 Cannot have = Sign.

H_0: =	H_0: ≤	H_0: ≥
H_1: ≠	H_1: >	H_1: <
TTT	RTT	LTT

Key words for H_0:
   is, equal, Same, at least, at most

Key words for H_1:
   is not, no equal, different, more than, less than, below, above, exceed, greater than

Feb 13-5:02 PM

I claim 10% of all students smoke.

H_0: $P = .1$ claim

H_1: $P \neq .1$ TTT

College claims the mean age of all students is at most 32.5 Yrs. μ

H_0: $\mu \leq 32.5$ claim

H_1: $\mu > 32.5$ RTT

Dept. claims that stand. dev. of all final exam scores is below 10. σ

$\sigma < 10$

H_0: $\sigma \geq 10$

H_1: $\sigma < 10$ claim, LTT

To determine the type of testing
Look at H_1

H_1 $\neq$ TTT

H_1 $<$ LTT
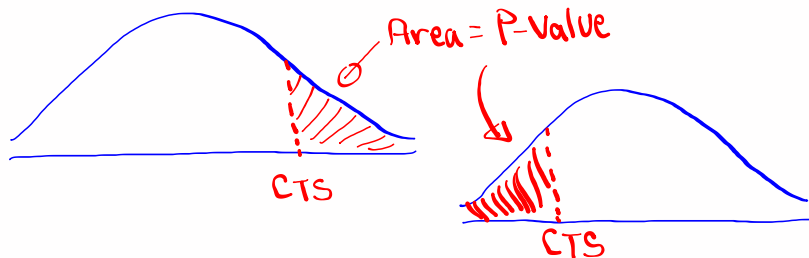
H_1 $>$ RTT

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What is P-Value?
P-Value is the area of the tail marked by CTS.
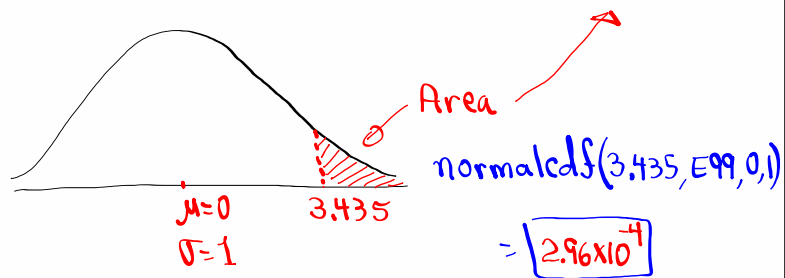
Only multiply by 2 if doing TTT.
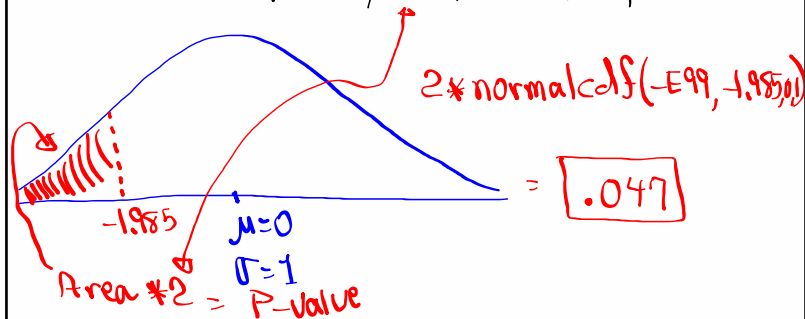
Area = P-Value

CTS

CTS

Multiply by 2 when working with TTT

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CTS $Z = 3.435$ RTT   Find p-value.

Area

normalcdf$(3.435, E99, 0, 1)$

$= \boxed{2.96 \times 10^{-4}}$

$\mu = 0$ 3.435
$\sigma = 1$

Given $Z = -1.985$, TTT   Find P-value

$2 * \text{normalcdf}(-E99, -1.985, 0, 1)$

$= \boxed{.047}$

-1.985 $\mu = 0$
$\sigma = 1$

Area $*2$ = P-value

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Given   CTS $t = 1.785$, RTT, $df = 9$

Find p-value

Area = P-value

tcdf$(1.785, E99, 9)$

$\mu = 0$ 1.785
σ unknown

$= \boxed{.054}$
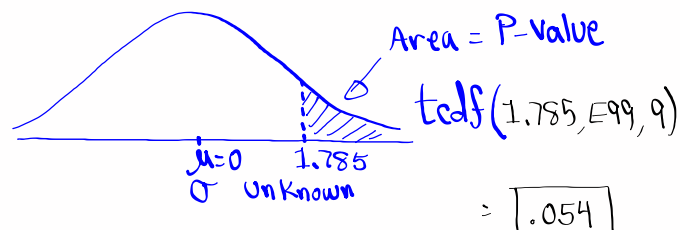
Given   CTS $t = -2.125$, $df = 14$, TTT

Find P-value

P-value $= 2 * $ Area

$= 2 * \text{tcdf}(-E99, -2.125, 14)$

-2.125 $\mu = 0$
σ unknown

$= \boxed{.052}$

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Four Possible outcomes for H_0:

H_0 / Action	Valid	Invalid
Fail-to Reject	✓	Type II error
Reject	Type I error	✓

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More on type of testing:

H_0 NCR $1-\alpha$ H_1 CR α CV   RTT

H_1 CR α CV H_0 NCR $1-\alpha$ LTT

H_1 CR $\alpha/2$ CV H_0 NCR $1-\alpha$ H_1 CR $\alpha/2$ CV   TTT
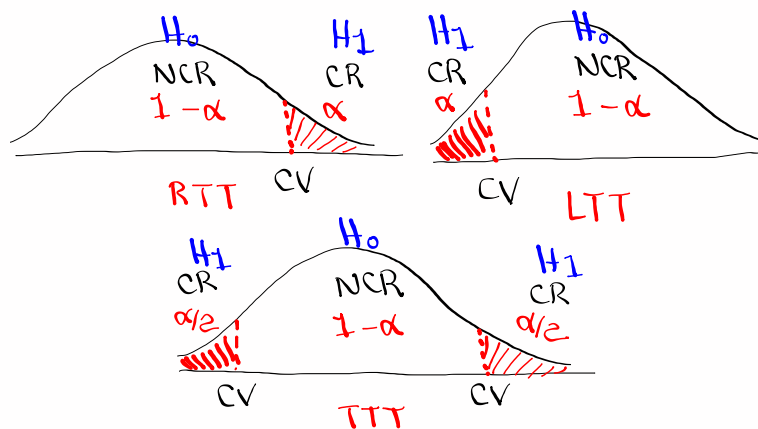
$$P(H_0 \text{ Valid}) = 1 - \alpha = P(H_1 \text{ invalid})$$

$$P(H_0 \text{ invalid}) = \alpha = P(H_1 \text{ Valid})$$

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Testing One Population Proportion P:

$H_0: P = P_0$ ⎫ $H_0: P \geq P_0$ ⎫ $H_0: P \leq P_0$

$H_1: P \neq P_0$ ⎬ $H_1: P < P_0$ ⎬ $H_1: P > P_0$

  TTT ⎭   LTT ⎭   RTT

CV Z, use invNorm

CTS Z
P-value P ⟹ 1 - Prop Z Test

Proceed with testing chart

Final Conclusion about claim.

Feb 13-5:49 PM

I claim 10% of all students smoke.

In a survey of 250 students, 35 were smokers.

use $\alpha = .02$ to test the claim.

$H_0: P = .1$ — claim

$H_1: P \neq .1$ TTT

CV Z TTT $\alpha = .02$

CTS $Z = 2.108$
P-value P = .035

1 - Prop Z Test
$P_0 = .1$ H_0
$x = 35$
$n = 250$
Prop $\neq P_0$ H_1
Calculate

H_1 CR .01 H_0 NCR .98 H_1 CR .01
-2.326 $\mu = 0$ $\sigma = 1$ 2.326
$Z = $ invNorm $(.99, 0, 1)$
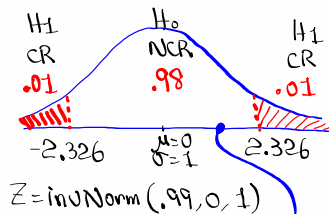
CTS is in NCR
H_0 Valid H_1 invalid
P-value $> \alpha$ ↘ valid claim
.035 .02   FTR the claim

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I claim that at least 40% of all students work while going to school. → $P \geq .4$

In a Survey of 400 Students, $n=400$ $x=.38(400)=152$ 38% of them were working.

$H_0: P \geq .4$ claim

Test the claim. NO α

Use α = .05

$H_1: P < .4$ LTT

CR H_1 NCR H_0
.05 .95

−1.645 $\mu=0$ $\sigma=1$

$Z = invNorm(.05, 0, 1)$

CTS $Z = -.816$

P-Value $P = .207$

1 − PropZTest
$P_0 = .4$ H_0
$x = 152$
$n = 400$
Prop $< P_0$ H_1
Calculate

CTS is in NCR

H_0 valid H_1 invalid

P-value $> α$ → valid claim
.207 .05

FTR the claim

AT&T claims that more than 75% of all students have iPhone. $P > .75$

I surveyed $n=450$ 450 students and 78% of them had iPhone. $x=450(.78)=351$

Use α = .01 to test the claim.

$H_0: P \leq .75$

$H_1: P > .75$ claim, RTT

CV Z RTT α = .01

H_0 H_1
NCR CR
.99 .01
2.326

$Z = invNorm(.99, 0, 1)$

CTS $Z = 1.470$

P-value $P = .071$

1 − PropZTest
$P_0 = .75$ H_0
$x = 351$
$n = 450$
Prop $> P_0$ H_1
Calculate

CTS is in NCR

H_0 valid H_1 invalid

P-value $> α$ Invalid claim
.071 .01

Reject the claim

I claim that majority of SF 49ers fan lost a bet.

$P > .5$ claim

In a survey of $\boxed{175 \text{ fans}}$ $\boxed{52\% \text{ of them}}$ lost a bet.

$\underline{n = 175}$ $x = 175(.52) = 91$

$H_0: P \leq .5$

$\boxed{\text{Test the claim.}}$

$H_1: P > .5$ claim, RTT

1 - Prop Z Test
  $P_0: .5$
  $x = 91$
  $n = 175$
  $\text{Prop} > P_0$ H_1
  $\boxed{\text{Calculate}}$

CTS $Z = .529$

P-Value $P = .298$

$\triangleright$ use $\alpha = .05$

P-Value $> \alpha$ H_0 valid
.298   .05 H_1 invalid

Invalid claim

Reject the claim

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